



## **AP<sup>®</sup> Calculus BC 2005 Free-Response Questions**

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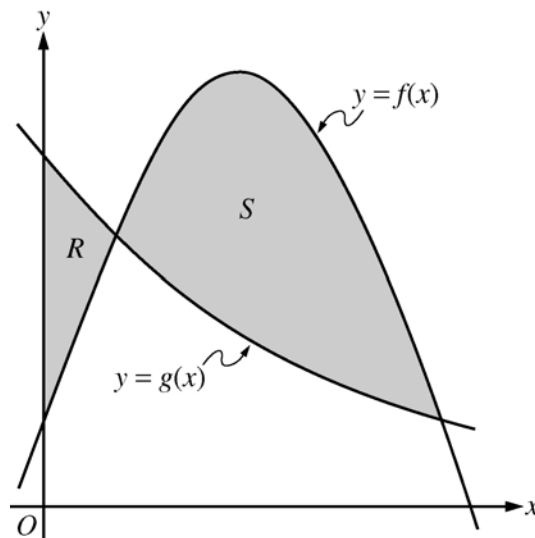
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**2005 AP<sup>®</sup> CALCULUS BC FREE-RESPONSE QUESTIONS**

**CALCULUS BC**  
**SECTION II, Part A**  
**Time—45 minutes**  
**Number of problems—3**

A graphing calculator is required for some problems or parts of problems.

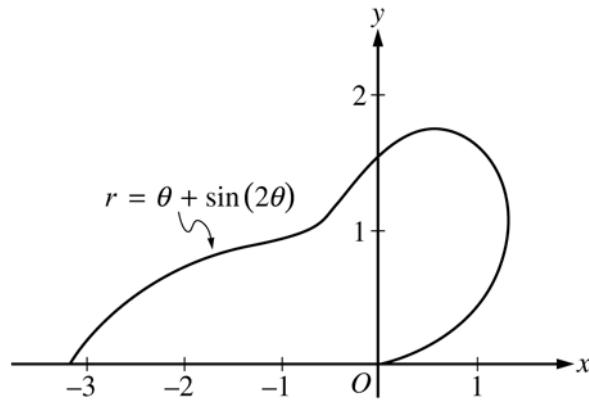
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1. Let  $f$  and  $g$  be the functions given by  $f(x) = \frac{1}{4} + \sin(\pi x)$  and  $g(x) = 4^{-x}$ . Let  $R$  be the shaded region in the first quadrant enclosed by the  $y$ -axis and the graphs of  $f$  and  $g$ , and let  $S$  be the shaded region in the first quadrant enclosed by the graphs of  $f$  and  $g$ , as shown in the figure above.
- (a) Find the area of  $R$ .
- (b) Find the area of  $S$ .
- (c) Find the volume of the solid generated when  $S$  is revolved about the horizontal line  $y = -1$ .
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2. The curve above is drawn in the  $xy$ -plane and is described by the equation in polar coordinates  $r = \theta + \sin(2\theta)$  for  $0 \leq \theta \leq \pi$ , where  $r$  is measured in meters and  $\theta$  is measured in radians. The derivative of  $r$  with respect to  $\theta$  is given by  $\frac{dr}{d\theta} = 1 + 2\cos(2\theta)$ .
- (a) Find the area bounded by the curve and the  $x$ -axis.
- (b) Find the angle  $\theta$  that corresponds to the point on the curve with  $x$ -coordinate  $-2$ .
- (c) For  $\frac{\pi}{3} < \theta < \frac{2\pi}{3}$ ,  $\frac{dr}{d\theta}$  is negative. What does this fact say about  $r$ ? What does this fact say about the curve?
- (d) Find the value of  $\theta$  in the interval  $0 \leq \theta \leq \frac{\pi}{2}$  that corresponds to the point on the curve in the first quadrant with greatest distance from the origin. Justify your answer.
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Distance $x$ (cm)	0	1	5	6	8
Temperature $T(x)$ ( $^{\circ}\text{C}$ )	100	93	70	62	55

3. A metal wire of length 8 centimeters (cm) is heated at one end. The table above gives selected values of the temperature  $T(x)$ , in degrees Celsius ( $^{\circ}\text{C}$ ), of the wire  $x$  cm from the heated end. The function  $T$  is decreasing and twice differentiable.
- (a) Estimate  $T'(7)$ . Show the work that leads to your answer. Indicate units of measure.
- (b) Write an integral expression in terms of  $T(x)$  for the average temperature of the wire. Estimate the average temperature of the wire using a trapezoidal sum with the four subintervals indicated by the data in the table. Indicate units of measure.
- (c) Find  $\int_0^8 T'(x) dx$ , and indicate units of measure. Explain the meaning of  $\int_0^8 T'(x) dx$  in terms of the temperature of the wire.
- (d) Are the data in the table consistent with the assertion that  $T''(x) > 0$  for every  $x$  in the interval  $0 < x < 8$ ? Explain your answer.
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**END OF PART A OF SECTION II**

**2005 AP<sup>®</sup> CALCULUS BC FREE-RESPONSE QUESTIONS**

**CALCULUS BC  
SECTION II, Part B  
Time—45 minutes  
Number of problems—3**

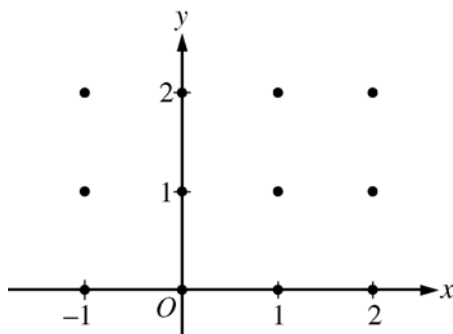
**No calculator is allowed for these problems.**

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4. Consider the differential equation  $\frac{dy}{dx} = 2x - y$ .

- (a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated, and sketch the solution curve that passes through the point  $(0, 1)$ .

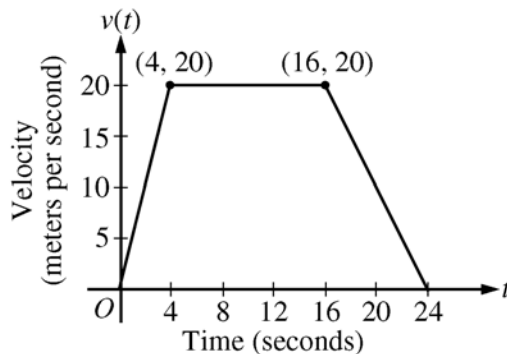
(Note: Use the axes provided in the pink test booklet.)



- (b) The solution curve that passes through the point  $(0, 1)$  has a local minimum at  $x = \ln\left(\frac{3}{2}\right)$ . What is the  $y$ -coordinate of this local minimum?
- (c) Let  $y = f(x)$  be the particular solution to the given differential equation with the initial condition  $f(0) = 1$ . Use Euler's method, starting at  $x = 0$  with two steps of equal size, to approximate  $f(-0.4)$ . Show the work that leads to your answer.
- (d) Find  $\frac{d^2y}{dx^2}$  in terms of  $x$  and  $y$ . Determine whether the approximation found in part (c) is less than or greater than  $f(-0.4)$ . Explain your reasoning.

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5. A car is traveling on a straight road. For  $0 \leq t \leq 24$  seconds, the car's velocity  $v(t)$ , in meters per second, is modeled by the piecewise-linear function defined by the graph above.
- (a) Find  $\int_0^{24} v(t) dt$ . Using correct units, explain the meaning of  $\int_0^{24} v(t) dt$ .
- (b) For each of  $v'(4)$  and  $v'(20)$ , find the value or explain why it does not exist. Indicate units of measure.
- (c) Let  $a(t)$  be the car's acceleration at time  $t$ , in meters per second per second. For  $0 < t < 24$ , write a piecewise-defined function for  $a(t)$ .
- (d) Find the average rate of change of  $v$  over the interval  $8 \leq t \leq 20$ . Does the Mean Value Theorem guarantee a value of  $c$ , for  $8 < c < 20$ , such that  $v'(c)$  is equal to this average rate of change? Why or why not?
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6. Let  $f$  be a function with derivatives of all orders and for which  $f(2) = 7$ . When  $n$  is odd, the  $n$ th derivative of  $f$  at  $x = 2$  is 0. When  $n$  is even and  $n \geq 2$ , the  $n$ th derivative of  $f$  at  $x = 2$  is given by  $f^{(n)}(2) = \frac{(n-1)!}{3^n}$ .
- (a) Write the sixth-degree Taylor polynomial for  $f$  about  $x = 2$ .
- (b) In the Taylor series for  $f$  about  $x = 2$ , what is the coefficient of  $(x-2)^{2n}$  for  $n \geq 1$ ?
- (c) Find the interval of convergence of the Taylor series for  $f$  about  $x = 2$ . Show the work that leads to your answer.
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**WRITE ALL WORK IN THE TEST BOOKLET.**

**END OF EXAM**